Routes of Learning: Highways, Pathways and Byways in the History of Mathematics
By Ivor Grattan-Guinness. Baltimore (The Johns Hopkins University Press). 2009. xii + 372 pp.

Grattan-Guinness' *Routes of Learning* is a compilation of articles that have appeared elsewhere. Such collections are often patchwork affairs, but surprisingly, this book does not suffer from any lack of coherence. This is likely due to the fact that Grattan-Guinness' work was motivated by a few key principles and seems to have been written in a consistent voice over the course of his long and productive career.

The book consists of 18 chapters, none of which are essays *in* the history of mathematics – that is, they do not discuss the mathematics of particular people or eras of the past. Rather, they are pieces *about* the history of mathematics: its practice, its philosophy, its history, its relationship with its neighboring disciplines and particularly its uses in mathematics education. The bookends are an introductory, autobiographical sketch and a closing bonbon. The 16 pieces in between are organized into three sections: Highways, Pathways and Byways. Highways – the largest group – consists of essays about the discipline of history of mathematics. The Pathways section concerns the uses of history in the teaching of mathematics. The Byways section concerns itself with mathematics and culture, specifically the relationship between mathematics and Christianity, as well as numerology and music theory.

By way of introduction, the first chapter is a brief autobiography, originally published in 1999. Here Grattan-Guinness describes the heuristic and historical questions that came to his mind when, as an undergraduate at Oxford, he was taught the very polished and "perfect" pure mathematics of the late 20th century: what was the motivation of these theories and how did they come about? His undergraduate teachers had no interest in these questions, but his own drive to answer them brought him to graduate studies in the history of science, and of mathematics in particular, after a brief career in the defense industry. In this piece, we learn about those individuals who influenced him in his early career, whether for good or ill. Grattan-Guinness also explains how he came upon his thesis topic (the development of analysis both before and after Cauchy) and his simultaneous interest in Fourier, as well as how these led him to the study of Cantor's development of set theory. Thus we come to appreciate how a rich career as a historian of analysis, mathematical physics and set theory was motivated by two key questions and developed, almost by logical necessity, though personal contacts during and shortly after graduate school.

The second chapter, which is the first essay in the Highways section, is "The Mathematics of the Past: Distinguishing Its History from our Heritage." This is the version that appeared in this journal in 2004; it is different from the version that was given as the Kenneth O. May lecture in 2002 [see Kinyon and Van Brummelen, 2005], which was geared more towards mathematics education. In both versions, Grattan-Guinness distinguishes between the history of mathematics, which addresses the question "What happened in the past?" from its heritage, which addresses the question "How did we get here?" The canonical example of the heritage point of view is the interpretation of Euclid's Proposition II.4 as a piece of "geometrical algebra," $(a + b)^2 = a^2 + 2ab + b^2$. In this essay, Grattan-Guinness explores the aspects and merits of each point of view and gives six examples of fields where "the conflation of history and heritage seems to be especially acute, even among historians." Not surprisingly, most of these examples are drawn from set theory, applied mathematics, and the foundation of calculus. He goes on to consider the value of each point of view

as a metatheory: "the issue is *not* history yes or no, but history how?" Here, as with many of the other pieces in this collection, Grattan-Guinness has updated the References section for this new edition. He also occasionally adds new material into his chapters, which he sets off with curly braces.

Among the other essays in the Highways section is a history of the History of Mathematics in the 20th Century, which includes both a survey by country as well as an examination of larger trends in the field (the title begins "Decline, Then Recovery"). The "Appraisals" portion of this essay includes a section titled "Cottage Industry or Ghetto," a provocative topic that segues nicely into the fifth of the Pathways essays, an examination of how the field seems "Too Mathematical for Historians, too Historical for Mathematicians." The Highways section also includes an examination of neglected features of history of mathematics, a look at general histories, a look at history of science journals, and finally, a "Skeptical Inquiry" into scientific revolutions. In this essay (and elsewhere) Grattan-Guinness describes his notion of "convolution" as more accurately describing the transitional period of "revolutionary" scientific activity.

The Pathways portion of *Routes of Learning* opens with an essay "On the Relevance of the History of Mathematics to Mathematics Education." It begins with a description of a course that Grattan-Guinness actually taught to masters students in mathematics education in Australia in 1976, followed by some general observations flowing from the specific case study. Those of us who teach similar courses thirty-five years later will stand in awe of what he managed to cover in one 10-week term! Here and in subsequent chapters in this section, those who teach history of mathematics to students majoring in mathematics or mathematics education will find valuable course material, both in specific detail and in general consideration. The other chapters in this section include paradoxes vs. contradictions, Euclid's approach to magnitudes and proportions, numbers and number systems, and an outline for a calculus course based on the historical development of the subject.

The final Byways section begins with two essays on the interactions between Christianity and mathematics. The first of these considers numerology, gematria and sacred geometry, not only in early and medieval Christian traditions, but also in the pre-Christian cultures. A second essay is concerned with links between Christianity and mathematics after 1750.

The final two essays in the Byways section, both with a musical theme, along with the closing essay, which Grattan-Guinness calls "Lollipops," are a sheer delight to read and ponder. The Lollipops are four "Pretty but Little-Known Theorems Involving the Triangle." Most of the Lollipops chapter was previously unpublished, except for the second theorem, previously published by Grattan-Guinness and apparently original to him, relating the arithmetic, geometric and harmonic mean of two magnitudes. The other three theorems, as well as a remarkable triangle theorem given earlier in the Byways section, all have long histories and yet are virtually unknown in our time.

This book assumes significant familiarity with the field of history of mathematics, so it may be out of reach of the general mathematical audience. However, it is a rich reference work for anyone teaching a history of mathematics course, and so deserves a place in most mathematics libraries. The early pieces contain much thought-provoking material about the history and philosophy of mathematics. But what of the later pieces, in the Byways section?

In the one of the last essays in *Routes of Learning*, Grattan-Guinness explores possible examples of numerology in the compositions of Mozart and Beethoven, and makes some general comments about music and numerology. Because he himself invoked Beethoven by name, I feel that I've been granted license to compare the later essays in *Routes of Learning* to Beethoven's Bagatelles. To paraphrase Bowen in the *Grove Dictionary of Music*, they

are typical of their author and show affinities with the greater books and essays that he was writing at the same time.

Reference

Kinyon, M., Van Brummelen, G. (Eds.), 2005. Mathematics and the Historian's Craft: The Kenneth O. May Lectures. CMS Books in Mathematics. Springer, New York.

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